Paper Reference (complete below)	Centre No.					Surname		Initial(s)
6 6 6 3 / 0 1	Candidate No.					**Signature**		

Paper Reference(s)

6663

Edexcel GCE

Core Mathematics C4
Advanced Subsidiary
Set A: Practice Paper 4

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
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Total	

Instructions to Candidates
In the boxes above, write your centre number, candidate number, your surname, initials and signature. You must write your answer for each question in the space following the question. If you need more space to complete your answer to any question, use additional answer sheets.

Information for Candidates
A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has nine questions.

Advice to Candidates
You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

Turn over

1.   The curve C has equation $5x^2 + 2xy - 3y^2 + 3 = 0$. The point P on the curve C has coordinates $(1, 2)$.

   (a) Find the gradient of the curve at P.

   (5)

   (b) Find the equation of the normal to the curve C at P, in the form $y = ax + b$, where a and b are constants.

   (3)

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2.   **Figure 1**
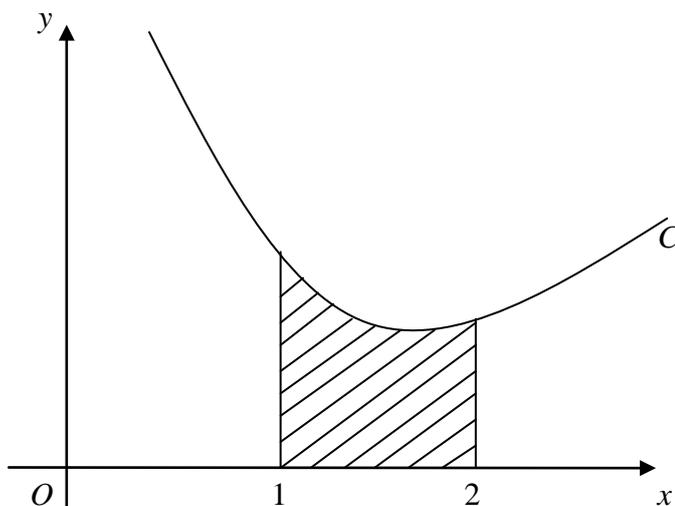
   In Fig. 1, the curve C has equation $y = f(x)$, where

   $$f(x) = x + \frac{2}{x^2}, \quad x > 0.$$

   The shaded region is bounded by C, the x-axis and the lines with equations $x = 1$ and $x = 2$. The shaded region is rotated through 2π radians about the x-axis.

   Using calculus, calculate the volume of the solid generated. Give your answer in the form $\pi(a + \ln b)$, where a and b are constants.

   (8)

2

3.
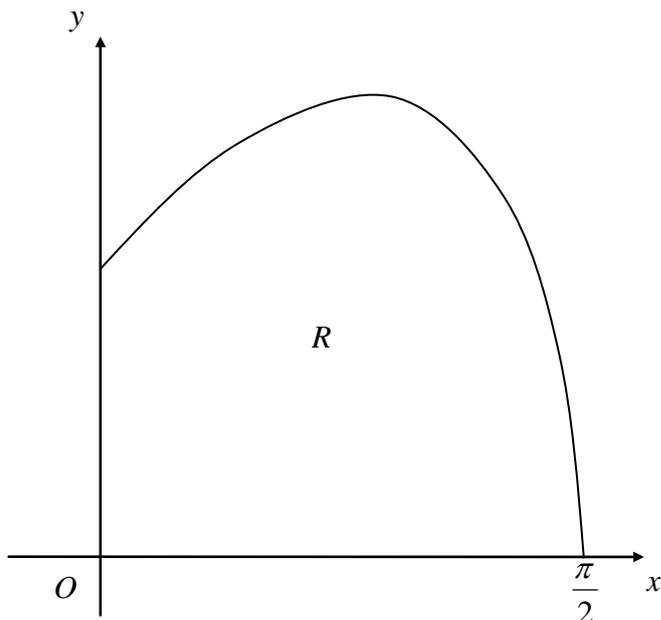
Figure 2

Figure 2 shows part of the curve with equation

$$y = e^x \cos x, \ 0 \leq x \leq \frac{\pi}{2}.$$

The finite region R is bounded by the curve and the coordinate axes.

(*a*) Calculate, to 2 decimal places, the *y*-coordinates of the points on the curve where $x = 0, \ \dfrac{\pi}{6}, \ \dfrac{\pi}{3}$ and $\dfrac{\pi}{2}$.

(3)

(*b*) Using the trapezium rule and all the values calculated in part (*a*), find an approximation for the area of *R*.

(4)

(*c*) State, with a reason, whether your approximation underestimates or overestimates the area of *R*.

(2)

3

4. A curve is given parametrically by the equations

$$x = 5 \cos t, \quad y = -2 + 4 \sin t, \quad 0 \le t < 2\pi.$$

(*a*) Find the coordinates of all the points at which *C* intersects the coordinate axes, giving your answers in surd form where appropriate.

(4)

(*b*) Sketch the graph at *C*.

(2)

P is the point on *C* where $t = \frac{1}{6}\pi$.

(*c*) Show that the normal to *C* at *P* has equation

$$8\sqrt{3}y = 10x - 25\sqrt{3}.$$

(4)

_____

5.   **Figure 1**
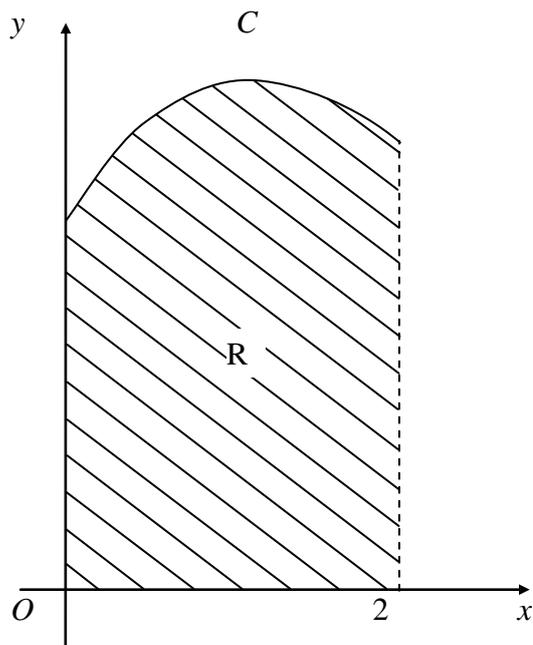
The curve C has equation $y = f(x)$, $x \in \mathbb{R}$. Figure 1 shows the part of C for which $0 \leq x \leq 2$.

Given that

$$\frac{dy}{dx} = e^x - 2x^2,$$

and that C has a single maximum, at $x = k$,

(*a*) show that $1.48 < k < 1.49$.

(3)

Given also that the point $(0, 5)$ lies on C,

(*b*) find f(x).

(4)

The finite region R is bounded by C, the coordinate axes and the line $x = 2$.

(*c*) Use integration to find the exact area of R.

(4)

5

6. When $(1 + ax)^n$ is expanded as a series in ascending powers of x, the coefficients of x and x^2 are -6 and 27 respectively.

(*a*) Find the value of a and the value of n.

(5)

(*b*) Find the coefficient of x^3.

(2)

(*c*) State the set of values of x for which the expansion is valid.

(1)

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7. Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines, l_1 and l_2, along which they travel are

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$
$$\text{and} \quad \mathbf{r} = 9\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mu(4\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ and μ are scalars.

(*a*) Show that the submarines are moving in perpendicular directions.

(2)

(*b*) Given that l_1 and l_2 intersect at the point A, find the position vector of A.

(5)

The point b has position vector $10\mathbf{j} - 11\mathbf{k}$.

(*c*) Show that only one of the submarines passes through the point B.

(3)

(*d*) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance AB.

(2)

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8. In a chemical reaction two substances combine to form a third substance. At time t, $t \geq 0$, the concentration of this third substance is x and the reaction is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(1 - 2x)(1 - 4x), \text{ where } k \text{ is a positive constant.}$$

(*a*) Solve this differential equation and hence show that

$$\ln \left| \frac{1 - 2x}{1 - 4x} \right| = 2kt + c, \text{ where } c \text{ is an arbitrary constant.}$$

(7)

(*b*) Given that $x = 0$ when $t = 0$, find an expression for x in terms of k and t.

(4)

(*c*) Find the limiting value of the concentration x as t becomes very large.

(2)

END